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University Endowment

How to Squander Your Endowment:
Pitfalls and Remedies

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Contents

1. An Overview of University Endowments

- An endowment is an aggregation of assets invested by a college or university to support its educational mission in perpetuity.

- Endowments serve institutions and the public by:
  - providing stability
  - leveraging other sources of revenue
  - encouraging innovation and flexibility
  - allowing a longer time horizon

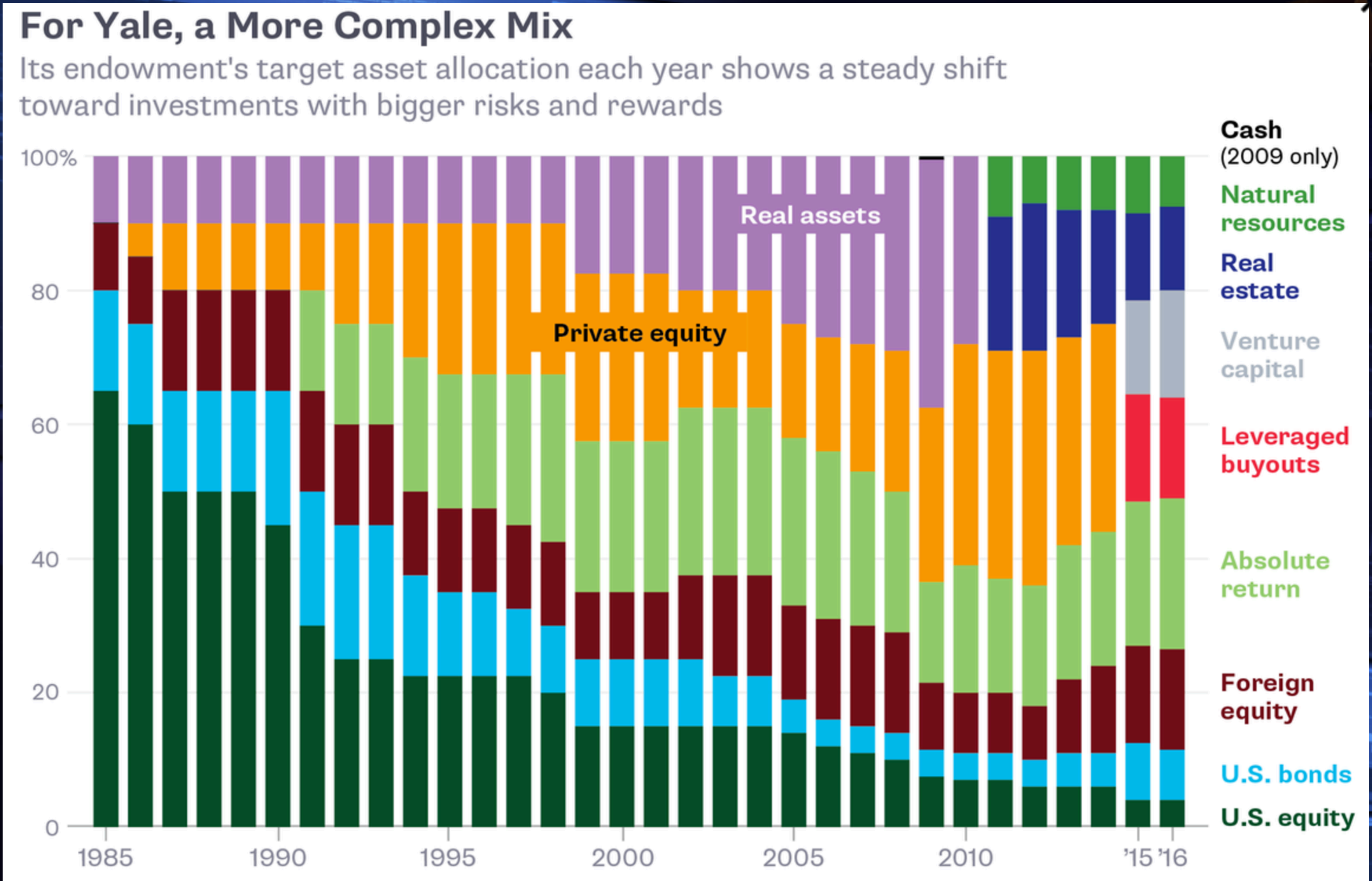
- Endowment funds follow very strict asset allocation policies and payout policies.

1. An Overview of University Endowments

Rank	Institution	State/ Province	2016 Endowment Funds ($000s)	2015 Endowment Funds ($000)	*Change in Market Value (%)
1	Harvard University	MA	34,541,893	36,448,817	-5.2
2	Yale University	CT	25,408,600	25,572,100	-0.6
3	The University of Texas System	TX	24,203,213	24,083,150	0.5
4	Stanford University	CA	22,398,130	22,222,957	0.8
5	Princeton University	NJ	22,152,580	22,723,473	-2.5
6	Massachusetts Institute of Technology	MA	13,181,515	13,474,743	-2.2
7	University of Pennsylvania	PA	10,715,364	10,133,569	5.7
8	The Texas A&M University System and Foundations[i]	TX	10,539,526	10,477,102	0.6
9	University of Michigan	MI	9,743,461	9,952,113	-2.1
10	Northwestern University	IL	9,648,497	10,193,037	-5.3
11	Columbia University	NY	9,041,027	9,639,065	-6.2
12	University of Notre Dame	IN	8,374,083	8,566,952	-2.3
13	University of California	CA	8,341,073	7,997,099	4.3
14	The University of Chicago	IL	7,001,204	7,549,710	-7.3
15	Duke University	NC	6,839,780	7,296,545	-6.3
16	Washington University in St. Louis	MO	6,461,717	6,818,748	-5.2
17	Emory University	GA	6,401,650	6,684,305	-4.2
18	University of Virginia	VA	5,852,309	6,180,515	-5.3
19	Cornell University	NY	5,757,722	6,037,546	-4.6
20	Rice University	TX	5,324,289	5,557,479	-4.2

**Data source is National Association of College and University Business Officers (NACUBO)*

1. An Overview of University Endowments



1. An Overview of University Endowments

- The most basic fiduciary responsibility of an endowment trustee is preservation of the corpus of the fund in perpetuity.*

- Two commonly used practices for endowments to preserve capital:

  1) Having a spending rate less than the expected return

     (E.g. "*The primary objective of the Great State University Endowment is to preserve the purchasing power of the endowment after spending...which means, on average, an annual total rate of return equal to inflation plus actual spending*".)

  2) Using a moving average rule to smooth spending

     (E.g. *UC Berkeyley, UC Irvine, and UC Santa Cruz plan to spend about 4.5% of a twelve-quarter (three year) moving average market value of the endowment pool*.)

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* Spitz, William. T. (1999), Investment Policies for College and University Endowments. New Directions for Higher Education, 1999: 51–59. doi:10.1002/he.10705

2.1 Definition of Preservation of Capital

- A policy preserves (resp. destroys) capital if the value of a unit of the endowment in real terms goes to infinity (resp. zero) over time in probability.
  - Returns are inflation−adjusted.
  - Future contributions are not included

- $W_t = W_{t-1}(1 + r_t - s_t)$
  - W_t: the real value of wealth in the unit at time t
  - r_t: the real rate of return at time t
  - s_t: the spending rate at time t

- Endowment wealth is said to be preserved if the real value of a unit W_t becomes arbitrarily large over time: $\text{plim}_{t\to\infty} W_t = \infty$.*
  Endowment wealth is said to be destroyed if the real value of a unit W_t vanishes over time: $\text{plim}_{t\to\infty} W_t = 0$.

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* As is conventional, *plim* indicates convergence in probability. By definition, $\text{plim}_{t\to\infty} W_t = \infty$ if for all $X > 0$, $\text{prob}(W_t > X) \to 1$ as $t\to\infty$

2.2 Preserving Capital in Discrete Time

❖ The traditional criterion says that the spending is less than the return on the portfolio, that is, s < r, then capital is preserved.

$$\begin{aligned} W_t &= W_{t-1}\,(1+r-s) \\ &= W_0\,(1+r-s)^t \end{aligned}$$

❖ However, the application of the law of large numbers is fallacious because the law of large numbers applies to sums, not products.

$$\begin{aligned} W_t &= W_{t-1}\,(1+r_t-s_t) \\ &= W_0\,\textstyle\prod_{i=1}^{t}(1+r_i-s_i) \end{aligned}$$

❖ E.g.: an endowment has a spending rate of 0% and an investment that triples or is reduced by a factor 1/9 with equal probabilities:

$$\begin{aligned} W_t &= W_0 \prod_{i=1}^{t}(1+r_i-s_i) \\ &= W_0 \exp\Big(\sum_{i=1}^{t}\log(1+r_i-s_i)\Big) \end{aligned}$$

$$1 \begin{cases} \nearrow 3 & \text{probability } 1/2 \\ \searrow 1/9 & \text{probability } 1/2 \end{cases} \Longrightarrow \frac{14}{9}$$

$$E[\log(1+r_i-s_i)] = \frac{1}{2}\log 3 + \frac{1}{2}\log\left(\frac{1}{9}\right) = \left(\frac{1}{2}+\frac{1}{2}*(-2)\right)\log 3 = -\frac{1}{2}\log 3 < 0$$

By the law of large numbers, $\mathrm{plim}\sum_{i=1}^{t}\log(1+r_i-s_i) = -\infty$ *and* $\mathrm{plim}W_t = 0$.

2.2 Preserving Capital in Discrete Time

- To correct the traditional criterion, we can convert the multiplication of a sum by taking logarithms:

$$\log(W_t) = \log(W_0) + \sum_{i=1}^{t} \log(1 + r_i - s_i)$$

- Recall $W_t / W_{t-1} = 1 + r_t - s_t$. Assume *1)* $W_0 > 0$, *2)* W_t / W_{t-1} .is i.i.d. over time, and *3)* $\log(W_t / W_{t-1})$ has finite mean and variance.

- Then, *1)* endowment capital is preserved if and only if $E[\log(W_t / W_{t-1})] = E[\log(1 + r_t - s_t)] > 0$ and *2)* endowment capital is destroyed if and only if $E[\log(W_t / W_{t-1})] = E[\log(1 + r_t - s_t)] < 0$.

- By Jensen's inequality and concavity of the logarithm, we have $E[\log(W_t / W_{t-1})] \leq \log(E[W_t / W_{t-1}])$ with inequality if and only if W_t / W_{t-1} is riskless.

- Corrected criterion $E[\log(W_t / W_{t-1})] = E[\log(1 + r_t - s_t)] > 0$

  Traditional criterion $E[(W_t / W_{t-1})] = E[(1 + r_t - s_t)] > 1$

2.2 Preserving Capital in Discrete Time

- E.g. Suppose a portfolio has a mean return of 5% and a standard deviation of 15%. The traditional rule says the mean spending rate must be less than 5%. By the Taylor series expansion, we have

$$\begin{aligned} E[\log(1+r-s)] &\approx E[r-s] - \left(\frac{1}{2}\right) Var[r-s] \\ &= 5\% - s - \frac{1}{2}(0.15)^2 \\ &= 3.875\% - s \end{aligned}$$

- Consider investing in a portfolio with risky asset.

  μ_p: mean return of risky asset

  θ: the proportion of risky asset in the portfolio

Traditional criterion: $\mathrm{r} + \theta(\mu_p - r) > s$.

Corrected criterion: the curvature of the logarithm implies that given s,

$$\mathrm{E}[\log(1 + \mathrm{r} + \theta(\mu_p - r) - s)] < 0.$$

2.3 Preserving Capital in Continuous Time

- The wealth of endowment follows the stochastic differential equation:

$$\frac{dW_t}{W_t} = \mu dt + \sigma dZt - sdt$$

- Applying Itˆo's Lemma to $\log(W_t)$, and using the above equation,

$$d\log(W_t) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dZt$$

$$\log(W_t) = \log(W_0) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma Zt$$

$$\mathrm{N}\left(\log(W_0) + \left(\mu - \frac{\sigma^2}{2}\right) t, \sigma^2 t\right)$$

- $prob(W_t \leq X) = prob(\log(W_t) \leq \log(X)) = N(\frac{\log(X) - \log(W_t) - (\mu - \frac{\sigma^2}{2} - s)t}{\sigma\sqrt{t}})$

$$\xrightarrow{t\uparrow\infty} \begin{cases} 0 & \text{if } s < \mu - \sigma^2/2 \\ 1/2 & \text{if } s = \mu - \sigma^2/2 \\ 1 & \text{if } s > \mu - \sigma^2/2 \end{cases}$$

3.1 Smooth Spending: Riskless Case

- A traditional moving average spending rule assumes the dynamic of spending to be

$$dS_t = \kappa(\tau W_t - S_t)dt$$

τ: the target spending rate

κ: the adjustment speed

- If the endowment only invests in a riskless bond with constant risk-free rate r, then the wealth process is given as $dW_t = rW_t dt - S_t dt$.

$$t = \frac{1}{\lambda_1 - \lambda_2} \ln\left(-\frac{K_2}{K_1}\right)$$

$$K_1 = \frac{W_0(\lambda_1 - \mathrm{r}) + \mathrm{S}_0}{\lambda_1 - \lambda_2} \text{ and } K_2 = \frac{W_0(r - \lambda_2) - \mathrm{S}_0}{\lambda_1 - \lambda_2}$$

$$\lambda_1 = \frac{r - \kappa + \sqrt{(\kappa - r)^2 - 4\kappa(\tau - r)}}{2} \text{ and } \lambda_2 = \frac{r - \kappa + \sqrt{(\kappa + r)^2 - 4\kappa\tau}}{2}$$

- Given a high initial spending rate S_0/W_0, the value of a unit declines proportionately more than spending. As the ratio of wealth to spending falls, this effect accelerates and wealth converges to zero in a “death spiral”.

3.1 Smooth Spending: Riskless Case

- Example:

  Assume $W_0 = 100$, $S_0 = 15$, $r = 5\%$, the target spending rate $\tau = 4\%$, and the adjustment rate $\kappa = 20\%$ each year.

  The wealth at the next year is

  $$W_1 = W_0\,(1 + r - s) = 100 * (1 + 5\% - 15\%) = 90$$

  The adjustment of spending is

  $$\Delta S = 20\% * (4\% * 100 - 15) = -2.2$$

  The spending rate in the next year:

  $$S_1 = (20 - 2.2) / 90 = 19.8\%$$

3.2 Smooth Spending: Risky Case

- Given the moving average spending rule, the endowment has return with constant mean and volatility, then the wealth process is given as

$$dW_t = W_t\,(\mu dt + \sigma dZ) - S_t dt = (W_t\,\mu - S_t)dt + W_t\,\sigma dZ$$

- Theorem: *when the endowment uses the moving average spending rule with positive target spending rate τ, no matter how small, and the i.i.d. investment process, the value of a unit hits zero in finite time (almost surely) and therefore capital is always destroyed.*

- Sketch of Proof
    1. Write down dynamics of $U_t = W_t / S_t$
    2. Find F increasing such that $Q_t = F(U_t)$ is a local martingale.
    3. Note that $F(0)$ is finite and $F(\infty) = \infty$.
    4. Since Q_t is a continuous local martingale, it is a time-changed Wiener process, i.e. there exists B_s with $B_0 = Q_0$ and unit variance per unit time with $Q_{v(s)} = B_s$ for some increasing continuous function v.
    5. We know B_s reaches 0 in finite time and we know how long B_s spends on average at different levels before hitting 0.

3.3 A Modified Smooth Spending Rule

❖ A modified smooth spending rule that preserves capital is:

$$dS_t = S_t \left(\underbrace{\kappa \left(\log\tau - \log\left(\frac{S_t}{W_t}\right)\right)}_{\text{Smooth spending with target } \tau} + \underbrace{\mu - \sigma^2/2 - \frac{S_t}{W_t}}_{\text{Adjusting for over-spending}} \right) dt,$$

❖ This smooth spending rule preserves capital if and only if the parameters satisfy the following condition:

$$\mu - \frac{\sigma^2}{2} - \exp\left[log\tau + \frac{\sigma^2}{4\kappa}\right] > 0$$

4. General Condition for Preservation of Capital

- Suppose Z is a standard Wiener process, the wealth dynamic follows

$$dW_t = Wt(\mu_t dt + \sigma_t dZ) - S_t dt = (W_t \mu_t - S_t) dt + W_t \sigma_t dZ$$

$$W_t = W_0 \exp\left[\int_{v=0}^{t} \left(\mu_v - \frac{1}{2}\sigma_v{}^2 - s_v\right) dv - \int_{v=0}^{t} \sigma_v dZ_v\right]$$

- Given some general stochastic processes of μ_v, $\sigma_v{}^2$, and s_v, and for $\forall v > 0$, $\sigma_v > 0$, and $s_v > 0$. and the following limit exist

$$\lim_{t\to\infty} \frac{1}{t} E\left[\int_{v=0}^{t} \left(\mu_v - \frac{1}{2}\sigma_v{}^2 - s_v\right) dv\right] = B$$

$$\lim_{t\to\infty} \frac{1}{t^2} Var\left[\int_{v=0}^{t} \left(\mu_v - \frac{1}{2}\sigma_v{}^2 - s_v\right) dv - \int_{v=0}^{t} \sigma_v dZ_v\right] = 0$$

then the spending process preserves capital in the sense that

$$\lim_{t\to\infty} \Pr(W_t < W_0) = 0,$$

if and only if the limit $B > 0$.

4. General Condition for Preservation of Capital

❖ *Special Case*: Temporarily Negative Real Risk-Free Rate

The stock price follows a diffusion process as

$$\frac{dP_t}{P_t} = (r_t - \iota + \pi)dt + \sigma dZ_t$$

where ι is a constant inflation rate and π is a constant risk premium. With a fixed portfolio θ in stock, the wealth process follows,

$$\begin{aligned} dW_t &= (r_t - \iota)W_t dt + W_t\theta(\sigma dZ_t + \pi dt) - S_t dt \\ &= W_t((r_t - \iota + \theta\pi)dt + \theta\sigma dZ_t) - S_t dt \end{aligned}$$

Then, the endowment can preserve capital if and only if

$$E\left[r_t - \iota + \theta\pi - \frac{\theta^2\sigma^2}{2}\right] > E[s_t]$$

where the spending rate s_t is covariance-stationary process.

5. Optimization Models

- Impose a drawdown constraint introduced by Grossman and Zhou (1993), which requires that wealth can never fall below a certain percentage of the previous maximum of wealth.

- Given the initial wealth W_0 and initial spending S_0, choose an adapted portfolio $\{\theta_t\}_{t=0}^{\infty}$ and an adapted rate-of-change-of-spending process $\{\delta_t = S'_t\}_{t=0}^{\infty}$ to maximize expected utility

$$\max_{\theta,\delta} E\left[\int_{t=0}^{\infty} e^{-\rho t} \frac{{S_t}^{1-R}}{1-R} dt\right]$$

$$s.t.\, dW_t = rW_t dt + \theta_t\big((\mu_P - r)dt + \sigma_P dZ_t\big) - S_t dt - k\frac{{\delta_t}^2}{S_t},$$

$$dS_t = \delta_t dt.$$

$$\forall t, Wt \geq 0.$$

Where ρ is the pure rate of time preference, and R is the constant relative risk aversion. It is assumed that $\mu_P - r$, ρ, σ_P, k, and r are all positive constants.

6. Conclusion & Personal Thoughts

- The paper provides two valid arguments on two commonly used rules of thumb to preserve capital for university endowment.
    1) Having a spending rate less than the expected return on assets
    2) Using a moving average rule to smooth spending

- The optimization program is incomplete and may be less useful for practitioners.

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